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REPRESENTATION AND CALCULATION OF THE SPECTRAL DENSITY OF THE FIELD OF ATMOSPHERIC TURBULENCE BASED ON THE RESULTS OF AIRCRAFT FLIGHT TESTS

by Raymond André and Albert Jouan

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Translation of "Représentation et calcul de la densité spectrale du champ de turbulence atmosphérique à partir d'essais en vol sur avion"

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ABSTRACT

This article presents a method of investigating the field of atmospheric turbulence based on the results of high-speed flights in a turbulent atmosphere.

The essence of the method consists in calculating the spectral density of the correlation functions characterizing the turbulence by way of the response of the aircraft regarded as a measuring instrument.

In Part 1 the physical assumptions and the mathematical model that will best fit the phenomenon are discussed.

In Part 2 a semi-analog analysis chain giving the spectral density of a stationary random process is described.

I. Introduction

Atmospheric turbulence and its significance for aeronautics are matters with which engineers and designers in all countries have been concerned for more than thirty years. A knowledge of atmospheric turbulence is essential to provide designers with standards for calculating the maximum stresses to which an aircraft may be subjected during its service life and for proportioning the structure. Atmospheric turbulence will likewise affect the rate of aging of the airframe (fatigue), the flight characteristics of the aircraft (maneuverability), and the "comfort" - in the widest sense - of passengers and crew.

This problem - which has been the subject of many previous works on the theoretical and the experimental level - is now especially acute in view of contemplated flight programs for:

a) military aircraft (full supersonic low-altitude operation);

b) the supersonic commercial transport (transonic landing approach, crossing "jet stream" zone at high altitude at supersonic speeds).

II. General

The phenomenon of atmospheric turbulence may be defined as the rapid variation in time and space of the velocity vector linked with the stream line, relative to an established steady-state regime which is itself a function of the average wind intensity. Further, these rapid variations involve an entirely random process, which distinguishes them from the phenomena of classical acoustics.

One approach to the study of turbulence is to fly an aircraft in a turbulent atmosphere and investigate its response. This method employs the aircraft as a peculiarly adaptable, direct measuring instrument.

Solution of the general problem of the response of a given aircraft type flying through a medium liable to turbulence involves several distinct steps.

1. The first step is to find the entry function of the system, that is, the "atmospheric turbulence field." Given the random character of the phenomenon, this implies a statistical description of the atmosphere, taking account of numerous parameters (altitude, temperature, nature of the ground, wind, humidity), and the analytic formulation of its geometric spatial properties.

2. The second step is to calculate the aerodynamic forces associated with the turbulent field (gust forces). The solution of this step, of course, depends on the results of the first.

3. The third step is to determine the transfer functions relating the motion of the aircraft to the gust forces. This step involves use of the laws of flight mechanics and a knowledge of the elastic behavior of the structure at the flying speed in question.

4. Finally, the last step combines the entry function with the transfer functions to obtain the final functions of aircraft motion, strains and stresses.

In the experimental procedure, the logical order set forth above is reversed:

- Firstly, the response of the aircraft is known (step 4).

- Step 3 is obtained by conventional techniques of testing stability and harmonics in flight.

- Step 2 requires simplifying hypotheses to be made a priori (their validity to be confirmed a posteriori) concerning: the coefficients to be introduced (for example, stationary or quasi-stationary); distribution over the span (for example, constant).

- Step 1 represents the final object of the test.

The purpose of this report is to describe a means of arriving, by aircraft flight tests, at an accurate knowledge of the atmospheric turbulence field using a spectral method of representation.

In Part One, we shall present - on the basis of existing theories - the physical and mathematical model that will best "fit" the phenomenon and specify the assumptions made and their consequences, for later experimental verification.

In Part Two, we shall describe a chain of analysis in which each link corresponds precisely to a known mathematical operation, giving the spectral density of the correlation functions of a stationary random process.

PART ONE

I. Tensor Representation of Atmospheric Turbulence Field

1. Definitions

Relative to a system of axes in straight-line translational motion linked with the established steady-state regime, let the turbulent velocity vector be

$$\bar{U}_\alpha(x_1 x_2 x_3 t) \text{ at a point } M,$$

$$\bar{U}_\beta(x'_1 x'_2 x'_3 t') \text{ at a point } M'.$$

The tensor $\bar{R}_{\alpha\beta} = \bar{U}_\alpha(x_1 x_2 x_3 t)\ \bar{U}_\beta(x'_1 x'_2 x'_3 t')$ representing the mean value of the product of the components defines the spatiotemporal velocity correlations linked with the fluid.

This tensor has 9 space and time components.

The basic problem is as follows: To find the analytic form of $\bar{R}_{\alpha\beta}$ for a given fluid as a function of the absolute position of point M and the relative position of M and M′ in the space-time domain.

For this we have the Navier-Stokes equations describing the evolution in a fluid of disturbances originating at great distances.

$$\frac{\partial U_\alpha}{\partial t} + \lambda\beta \frac{\partial U_\alpha}{\partial x_\beta} + \frac{\partial}{\partial x}(U_\alpha U_\beta) = -\frac{\partial P}{\partial x_\alpha} + \nu \frac{\partial^2 U_\alpha}{\partial x_\alpha, \partial x_\beta} \qquad (1)$$

These partial differential equations call for the following remarks:

a) They are valid for an incompressible fluid;

b) They are not linear;

c) They involve multiple correlations of the velocity components;

d) Their boundary conditions are not known.

They are not solvable. To simplify the problem, we must introduce some new assumptions about the structure of the single correlations and their relationships with the double correlations, themselves based on certain invariance and symmetry properties of the space and time tensor $R_{\alpha\beta}$.

2. Assumptions

Homogeneity

The turbulence field will be said to be homogeneous if the components of the double tensor $R_{\alpha\beta}$ are functions solely of the five independent variables

$$\xi_1 = x_1 - x_1', \quad \xi_2 = x_2 - x_2', \quad \xi_3 = x_3 - x_3', \ t, \ t'.$$

It would not be physical to postulate homogeneity of turbulence throughout the spatial domain D, but it is possible to consider sub-domains d of sufficient extent that do possess this property.

Isotropy

The turbulence field will be said to be isotropic if the components of the tensor $\bar{R}$ are invariant with respect to rotations or permutations of the coordinate axes. This leaves only the three variables

$$r = \sqrt{\xi_1^2 + \xi_2^2 + \xi_3^2}, \ t, \ t'.$$

These two assumptions postulate properties of invariance and symmetry of the pure spatial components of the tensor R, the second assumption complementing the first in the sense that the symmetry is spherical.

Stationary State

The turbulence field will be said to be stationary (or homogeneous in time) if the same tensor components depend only on two variables

$$r, \ \tau = t' - t.$$

Physically, for an imperfect fluid, this assumption is incompatible with the assumption of homogeneity even for a spatial subdomain d of sufficient extent. For if the kinematic viscosity ν is not zero, the phenomenon will not be conservative in time, owing to the dissipation of energy in the form of heat.

Conversely, if thanks to a steady (but spatially localized) energy supply the phenomenon is locally stationary, the intensity of the turbulence will be spatially damped, and there will not be homogeneity.

3. Models of Turbulence

On the basis of these various assumptions, three models of turbulence were devised, and more or less satisfactorily verified by experiments - wind tunnel and free flow -, in view of the enormous difficulties of an experimental approach to the measurement of the crosscomponents of the tensor involving both space and time.

1) Turbulence with spherical symmetry; homogeneous, isotropic, but not stationary (Heisenberg).

2) Turbulence with cylindrical symmetry; about a principal axis defined by the mean fluid flow; homogeneous within a limited domain, nonisotropic, stationary. This system seems especially suitable for wind tunnel tests.

3) Axially symmetric turbulence (Batchelor, 7); homogeneous, nonisotropic, nonstationary.

II. Spectral Description of Atmospheric Turbulence

1. Spectral Description of Turbulence Field Introduced by G. I. Taylor in 1938

It will be seen that this description is highly serviceable because - by way of the notion of an energy spectrum - it introduces an additional degree of freedom in the determination of subdomains - not only with respect to space but also with respect to time -, the two parameters being linked by Taylor's hypothesis.

Taylor's Hypothesis

This consists in linking the pure time correlation function with the pure space correlation function along a preferred axis defined by the mean velocity vector V of the fluid, or moving body, in accordance with the relation

$$r = \vec{V} \times \tau.$$

Thanks to this hypothesis, the spectral tensor $\psi_{\alpha\beta}$ can now be defined in terms of the Fourier integral by the relation

$$R_{\beta\alpha}(\xi, t, t') = \int_{-\infty}^{+\infty} e^{i(\xi k + t'q' - tq)} \psi_{\alpha\beta}(k,q,q') \, dk.dq.dq' \qquad (2)$$

with ξ, k vectors; q, q' frequencies; dk element of volume in the domain of the wave numbers.

2. Heisenberg Has Shown That Given Isotropic Turbulence the Spectral Tensor Depends on Single Scalar Function $E_0(k,t)$

This function, which has a simple analytic form, possesses the valuable property of being real and positive, and plays the part of an energy density.

Proceeding from this idea, the mean energy of the turbulence may be defined by the integral

$$\frac{1}{2}\sum \bar{U}_\alpha^2 = \int_0^\infty E_0(k,\ t) \cdot dk. \tag{3}$$

Experimentally, it is important to know two components of the spectral tensor:[1]

1. Spectral space-time longitudinal correlation (or autocorrelation) function.

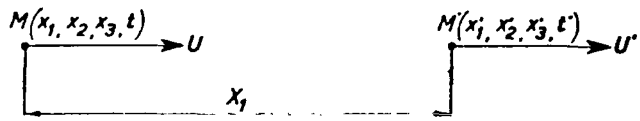

Fig. a.

Knowledge of this function tells us the structure of the turbulence along the preferred axis defined by the flight path of the aircraft.

2. Spectral transverse space correlation function.

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[1]Flight tests do not give direct data on the correlation functions of the U component of the spectral tensor directed along the principal axis, but on the correlation functions of the W component perpendicular to U in the vertical plane defined by the points M and M'.

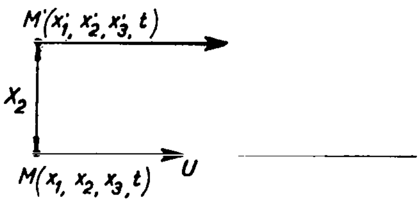


Fig. b.

Knowledge of this function tells us the structure of space in the horizontal plane, perpendicular to the principal direction, thus enabling us to verify the isotropy.

3. Schematization of the Aircraft Regarded as a Measuring Instrument (Fig. 1)

The aircraft is schematized as a flat plate, of small thickness and finite aspect ratio, moving through the atmosphere at a constant speed V and at a substantially constant altitude above the group (ρ constant).

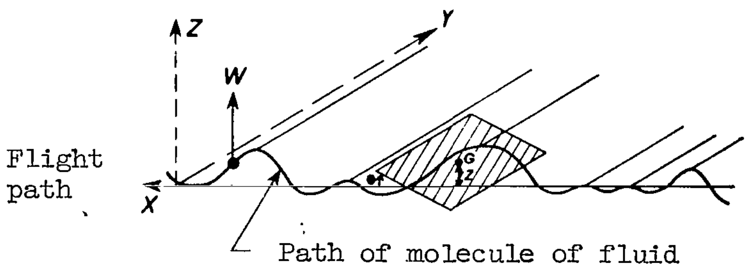


Fig. 1.

This plate is capable of moving bodily about its center of gravity, as defined by $Z'(\omega, t)$, the vertical velocity of the center of gravity, $\theta'(\omega, t)$, the rate of variation of the attitude of the plate. The plate is also subject to elastic strains defined by the vertical velocity $Z'(\omega, t)$ at any point.

The span of the plate is regarded as small enough for the Y-variations of the turbulence field to be regarded as negligible, i.e. $\partial W/\partial Y = 0$, where W is the vertical velocity of the fluid molecules.

This basic test is very important. Its analysis gives the spectral auto-correlation function of the vertical component W of the turbulence in terms of the response of the aircraft and permits verification of the assumptions of spatial homogeneity and especially stationariness.

4. Scheme of Representation of Spectral Density of Auto-Correlation Function

Many authors have made spectral analyses of the recorded motion of an aircraft flying through gusts - Zbrozek, Riedland, Panofsky (6 and 9) abroad; in France, the Brétigny Flight Test Center (5), Sud-Aviation and O.N.E.R.A. have begun to analyze Mystére IV A flight tests. The spectra have the following general form (Fig. 2).

The curve has four distinct zones.

Zone A: $0 < N <$ few tenths of a cps.

This zone of very low frequencies is very difficult to reach experimentally because of possible drift of the measuring instruments.

It is believed to be a zone of energy input with very long wave lengths (several kilometers), influenced by thermal or mechanical phenomena (relief and wind).

In this zone, verification of the assmptions of stationariness and homogeneity scarcely makes sense.

However, it is worth knowing the limit of the spectrum for $\omega = 0$ in order to define the scale of turbulence.

Zone B: few tenths of a cps $< N <$ few cps.

Accurate knowledge of this zone is essential for three reasons:

1. It represents an extremely high energy level;

2. It contains the stability frequencies of the aircraft (for example, in pitch);

3. Last and most important, the law of decrease of the ψ_{ww} curve will serve to confirm or refute existing theories.

According to Kolmogorov (1961), this zone is supoosed to correspond to energy transfer without degradation (permissible to put

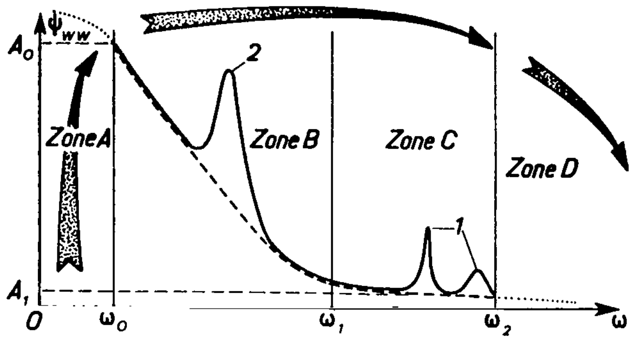


Fig. 2.

Zone A   Energy input
B, C   Energy transfer, $\nu = 0$
D   Energy degradation, $\nu \neq 0$

1   Structural mode frequencies
2   Pitch frequency

--   W spectrum, atmosphere
— Z' spectrum, aircraft
...   Not analyzed

$\nu = 0$ for these wavelengths). Under the influence of the nonlinear terms due to inertia forces, the component spectral lines are assumed to be in statistical equilibrium; this equilibrium would be largely independent of the initial conditions that produced the turbulence. In this zone, therefore, the curve should have a fairly universal form.

Moreover, the coefficient ν being zero, spatial homogeneity and stationariness should be verified simultaneously, as is possible for a perfect gas.

Zone C: few cps < N < few tens of cps.

1. It contains the structural frequencies of the aircraft;

2. Although the level is much lower than in zone B (ratio A_0/A_1 is at least of the order of 100), the overloading factors due to the elastic modes may bring about major amplifications. Examples: For low damping α = 10 per mil, we would have q = 100.

This zone also corresponds to energy transfer without degradation (ν = 0).

We analyzed this zone up to a frequency of N = 20 cps. The exponential decrease as a function of ω appears to have a definitely smaller exponent than in zone B.

Zone D: N > few tens of cps.

This zone is of interest only as qualifying the asymptotic behavior of the spectral curve. As the influence of the viscosity increases with frequency (short wavelengths), the convergence to zero should be rapid.

However, it must be borne in mind that the aircraft is too insensitive and imperfect an instrument for measurements in this frequency range.

5. Finding Spectral Function of W (Entry Function) from Spectral Function of Aircraft Response (see Fig. 2)

We shall say only a few words on this subject, which we have not yet attacked experimentally. It will be the subject of later publications after flight tests on the Nord 2508 B-01 at Villaroche.

Zone B: The Flight Test Center has already obtained some very good results in this zone by analog methods (cf. Study Report No. 653 by J. Perrochon). If $E(\omega)$ is the scalar representing the spectral density of the turbulence, $T^2(\omega)$ the square of the aircraft control transfer function, and $S(\omega)$ the spectral density of the aircraft response, we have in matrix form

$$[E(\omega)] = \frac{[S(\omega)]}{[T^2(\omega)]} \tag{4}$$

<u>Zone C</u>: In the range of elastic strains, equation (4) continues to hold. The transfer function $T'(\omega)$ can be obtained from a preliminary harmonic flight test.

The problem is much more complicated, however, because it is necessary to introduce the lag in the establishment of gust forces, a function of the low frequency, and take into account the distribution of turbulence over the span, which is not negligible at these wavelengths.

We shall attempt a quick check, based on Nord 2508 flight tests, at least on the first structural modes (bending, 2 nodes, torsion, fuselage).

PART TWO

SPECTRAL ANALYSIS OF STATIONARY RANDOM FUNCTIONS

I. General

Determining the spectrum of the response of a structure to a given citation, a certain function of time, is a classical problem.

In the case of spectral analysis of the response of an aircraft structure to the excitation generated by a turbulent fluid, the difficulty arises from the fact that the source of excitation is part of a random process.

We shall assume:

1. that the process is stationary;

2. that statistically it obeys the Laplace-Gauss law, and so depends solely on second-order moments.

It can be shown in this case that the process is completely defined by the "power" of the phenomenon, a function linked with the square of the amplitude and hence with the energy.

We saw in Part One that spectral representation of the space-time auto-correlation function of the atmospheric turbulence field and of the pure space intercorrelation function enables us to describe this field with sufficient accuracy.

Thus the rigorous plotting of the spectral density of the correlation functions is the crux of this method.

II. Methods of Calculating Spectral Density

Let the function $x(t)$ pertain to a stationary and ergodic random process. The auto-correlation function is then defined by (M = = mean value)

$$R_{xx}(\tau) = M\,[x(t) \cdot x(t + \tau)]$$

$$R_{xx}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{+T} x(t)\, x(t + \tau)\, d\tau. \qquad (6)$$

The spectral density or "power" will be given by the Fourier transform of $\underline{R}_{xx}(\tau)$,

$$P_{xx}(\omega) = \frac{1}{\pi} \int_{-}^{+\infty} R_{xx}(\tau) \cdot e^{-j\omega\tau} \cdot d\tau \qquad (7)$$

and since $\underline{R}_{xx}(\tau)$ is an even function,

$$P_{xx}(\omega) = \frac{2}{\pi} \int_{0}^{+\infty} R_{xx}(\tau) \cdot \cos \omega\tau \cdot d\tau. \qquad (8)$$

Now if we call $x_T(t)$ the function

$$x_T(t) = x(t) \text{ for } -T \le t \le T$$
$$0 \text{ for } t < -T,\ t > T \qquad (9)$$

and if $\underline{X}(j\omega)$ is the Fourier transform of $x(t)$,

$$X_T(j\omega) = \frac{1}{2\pi} \int_{-T}^{+T} x(t) \cdot e^{-j\omega t}\, dt \qquad (10)$$

and

$$P_{xx}(\omega) = \lim_{T \to \infty} \frac{\overline{X_T(j\omega)} \cdot X_T(j\omega)}{T} \quad \lim_{T \to \infty} \frac{\left| X_T(j\omega) \right|^2}{T} \tag{11}$$

The spectral density of the auto-correlation function can therefore be obtained in two different ways:

a) By a <u>purely mechanographic method</u>, using the auto-correlation function (6) and then taking the Fourier transform of that function. This is a long and expensive procedure, and for the upper parts of the spectrum it requires a very high mechanographic input capacity (10 points per half-period, or 400 readings per second to reach 20 cps).

b) By a <u>purely analog method</u> (selective filtering). Suppose we have an ideal crenelate type pass-band filter with transfer function

$$Z(j\omega) = 1 \text{ for } \omega_1 < \omega \leq \omega_2$$

$$0 \text{ for } \omega < \omega_1,\ \omega > \omega_2.$$

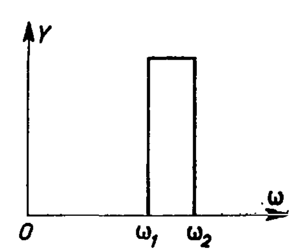


Fig. 3.

Between the Fourier transforms of the input and output functions we shall have the relation:

$$Y\ (j\omega) = Z\ (j\omega) \cdot X\ (j\omega). \tag{12}$$

The output power will be given by

$$\begin{aligned} \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{+T} y(t)^2 \cdot dt &= \int_0^\infty P_{yy}\ (\omega) \cdot d\omega \\ &= \int_0^\infty \frac{|\ Y\ (j\omega)\ |^2}{T}\ d\omega \end{aligned} \tag{13}$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_0^\infty |Z(j\omega) \cdot X(j\omega) \cdot X(j\omega)|^2 d\omega \tag{14}$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_{\omega_1}^{\omega_2} |X(j\omega)|^2\ d\omega. \tag{15}$$

hence

$$\int_0^\infty P_{yy}\ (\omega) \cdot d\omega = \lim_{T \to \infty} \frac{1}{T}\ |\ X\ (j\omega)\ |^2 \text{ for } \omega_2 \to \omega_1. \tag{16}$$

This method calls for three critical remarks.

1. The actual filter is never ideal; it has a self-attenuation α that is not zero. The mathematical Fourier transform

operation is therefore not exact, being performed not with respect to the $j\omega$ but with respect to the $\alpha + j\omega$ axis.

2. It will not permit analysis in amplitude and phase (real and imaginary parts). Hence the spectrum of the intercorrelation functions cannot be investigated by this method. In fact, for these functions we shall have

$$P_{xy}(\omega) = \frac{1}{T} \int_{-\infty}^{+\infty} R_{xy}(\tau) \cdot e^{-j\omega\tau} \circ d\tau \qquad (17)$$

$$P_{xy}(\omega) = \lim_{T \to \infty} \frac{\overline{X_T(j\omega)} \cdot Y_T(j\omega)}{T} \qquad (18)$$

3. Lastly, and this is the most serious objection, the final integration is generally carried out by means of an electric integrator, the graphical planimetering of the curve being an extremely time-consuming and tedious task.

Now the integration time T, representing the time constant of the electric integrator, must be far greater than the periods τ_i to be analyzed.

As the spectral measurement must be a mean of instantaneous values taken in the interval - T, + T, there is an inherent incompatibility between the definition of a correct mean value and of a precise point ω in the spectrum, an incompatibility between the determination of frequencies and times.

III. Semi-Analog Method of Spectrum Calculation

In this section we shall describe the random function spectral analysis chain of which a block diagram is given in Fig. 7.

It consists of:

1. An M 400 Tolana readout unit and demodulator delivering

the signal $\underline{X}(t)$ to be analyzed.

2. A Muirhead two-phase generator giving two harmonic references

$$\lambda \cos \omega t, \quad \lambda \sin \omega t.$$

This generator can go down to 1/100 cps.

3. A two-channel multiplier that can perform the following operations instantaneously:

Let $\underline{X}(t)$ be the random process, supposed to consist of a set of functions of the form

$$X\ (t) = \sum_{n=1}^{\infty} [a_n \cdot \cos n\ \Omega t + \sin n\ \Omega t]. \tag{19}$$

After multiplication,

$$X\ (t) \cdot \lambda \cos \omega t = \lambda a_n \cos \omega t \cdot \cos n\Omega t \tag{20}$$

and since the product vanishes for $\omega \# n\Omega$,

$$= \lambda \cdot a_n \cos^2 \omega t = \lambda a_n \left(\frac{1 - \cos 2\ \omega t)}{2} \right. \tag{21}$$

$$X\ (t) \cdot \lambda \cos \omega t = \left[\frac{\lambda a_n}{2}\right] - \text{harmonic } 2. \tag{22}$$

Similarly we have

$$X\ (t)\ \cdot\ \lambda \sin \omega t = \left[\frac{\lambda b_n}{2}\right] - \text{harmonic } 2. \tag{23}$$

It is not possible to suppress harmonic 2 by filtering without introducing large time constants. But the final integral will not be falsified; in fact,

$$\left.\begin{array}{l} \int_0^T \cos \omega t \cdot dt \to 0 \\ \\ \int_0^T \sin \omega t \cdot dt \to 0 \end{array}\right\rangle \text{ as } T \to \infty \tag{24}$$

4. An instantaneous numerical integrator, by frequency modulation and counting.

Fig. 4

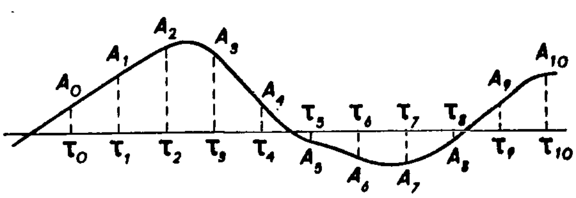


Fig. 5

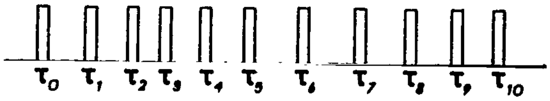


Fig. 6

Here are some details on the integrating process used, which is original so far as we know.

a. Required to integrate with respect to time, between two terminals τ_1 and τ_2, a continuous voltage, varying in amplitude and sign, put out by the multiplier.

Let us enlarge a very short interval taken out of Fig. 4.

After frequency modulation, the phenomenon will appear as follows.

The density of "crenelations" per second thus depends on the amplitude and sign of the low-frequency voltage modulating the carrier. Arithmetical counting of these crenelations in a Rochar decade counter (type A 1213) gives a reading proportional to the algebraic area included between the curve in Fig. 4 and the time axis, or in other words the integral of the signal.

In practice, preliminary counting of the central carrier with signal input short-circuited supplies a value N_0. Counting with input unshorted supplies a value N_1.

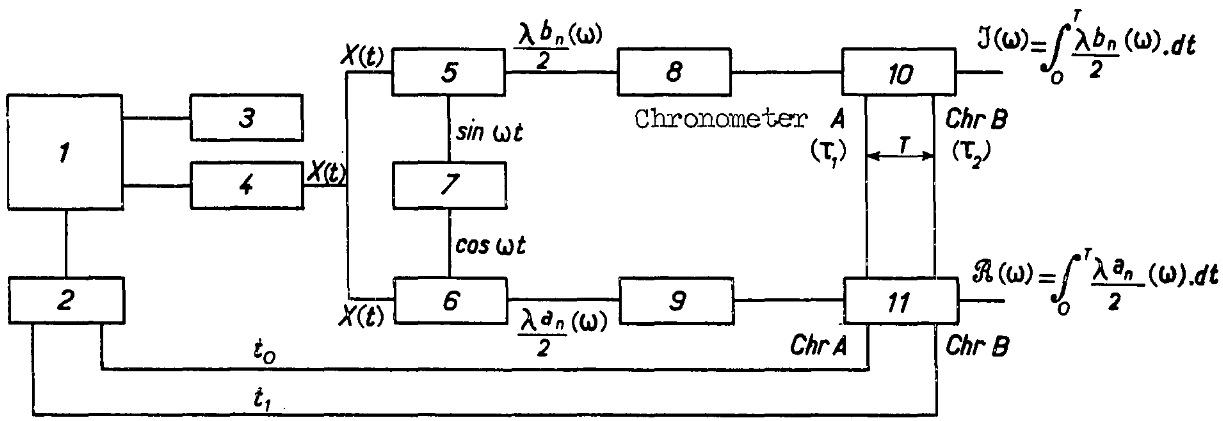
We have $I = |N_1 - N_0| \times (\text{sign of } N_1 - N_0)$.

b. The apparatus used comprises:

- Two tube modulators with central carrier N = 10,000 cps.

- Two Rochar decade counters with storage capacities of 10^6 units. The counting range thus covers 100 seconds, but this can be multiplied by shifting the first place.

- The accuracy of this method, using trapezoidal integration, is very high, in view of the large number of readings per



$$x(t) = \sum_{n=1}^{\infty} [a_n \cos n\Omega t + b_n \sin n\Omega t].$$

Fig. 7. Block diagram of analysis chain.

1. Readout unit
2. Sequencer, programmer
3. Audio
4. Demodulator
5. Multiplier 1
6. Multiplier 2
7. Two-phase VLF generator
8. Modulator 1
9. Modulator 2
10. Counter 1
11. Counter 2

second (10^4).

Sensitivity is high if the maximum modulation "swing," 40% of the central frequency is used; the Rochar count being defined within $\pm$ 1 unit, two fundamental properties of the equipment are necessary to use this method:

a. Perfect stability of modulators during measurement (drift less than 10^{-4} after warming up one half-hour);

b. Precise and constant definition, between each measuring point, of the interval

$$T = \tau_2 - \tau_1. \tag{26}$$

5. Programmer, sequencer. - The interval T is defined by a programmer built by the OR Electronics Laboratory.

a. A harmonic signal from a generator with very high frequency stability is pre-recorded on an auxiliary track of the analysis tape. By numerical counting in a 7-decade sequencer, it will determine the beginning (τ_1) and end (τ_2) of the interval T regardless of later variations in speed of the tape (chance variations in feed or high-speed run for extra-fast analysis).

b. At time τ_1 the programmer-sequencer furnishes a "top" which opens the starting switch of the Rochar counter (chronometer A). At time τ_2, it furnishes another "top" which closes the same switch (chronometer B).

6. Thus we have numerically the three quantities

$$T = \tau_2 - \tau_1$$

$$R\ (\omega) = \int_0^T X\ (t) \cdot \lambda \cos \omega t \cdot dt \qquad (27)$$

$$J\ (\omega) = \int_0^T X\ (t) \cdot \lambda \sin \omega t \cdot dt$$

whence by a simple arithmetical operation

$$P_{\omega\omega} = \frac{1}{T} [(R_{(\omega)})^2 + (J\ (\omega))^2] \qquad (28)$$

which is the function sought.

IV. Possible Automation of the Chain; Extra-Fast Analysis

Automation of the measurement of $\underline{R}(\omega)$ and $J(\omega)$ from a recopy will be obtainable in a matter of months. It will involve:

a) A "long loop" (100 meters) being developed by the Tolana firm, the purpose of which is not to render the phenomenon being analyzed periodic but simply to avoid having to "back up" the magnetic tape to analyze different ω_i's of the spectrum. A short circuit linked with part of the tape then permits step-by-step advance, with each cycle, of the generator.

b) Two ADDO X printing machines connected to and actuated by the Rochar counters.

c) Finally, by means of a preliminary recopy of the original, the feed rates can be multiplied by 16 (with no blurring to speak of).

A test flight lasting 16 minutes can be analyzed in 1 minute per frequency point. A complete spectrum of 100 points between 0 and 10 cps can thus be obtained in 2 hours.

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